Noisy Restaurants and Gene Expression

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This morning someone was complaining on TV that to go to a certain elegant restaurant you need to set up an appointment months in advance (or be famous enough to skip the line). I am not one to go to such restaurants myself, but I have often seen this phenomenon and wondered: if so many people want to go there, why don't they keep raising the menu prices, to the point where the demand meets their limited supply of tables?

One might argue that they don't want to offend clients' sensibilities by charging too much, or that it is good PR to have a long line of people waiting. But the incentive for a price increase must be quite strong, and there are other instances of limited supply (think Fortune 500 executive pay) where prices raise in the face of public outrage.

Another explanation might be the following. There is a positive feedback between the demand for a restaurant (how many people want to dine there in a given night) and its occupancy rate. That is, the more busy a restaurant is, the more people want to go there, and vice versa. This creates a bistable behavior: either the restaurant keeps full and everybody wants to go, or the place is empty and everyone is chased off (and the restaurant has to close). Also, the demand for a restaurant can suddenly change in response to varying fashions, a new restaurant in the corner, etc. That is, the demand has a noise component out of the restaurant owner's control.

By keeping the demand higher than is necessary to fill up the restaurant, the restaurant's owner makes it more unlikely that a relatively small change in demand will result in a switch from high to zero occupancy rate. This is because the additional demand acts as a buffer: if a sudden decrease of interest in this food were to reduce the demand, say, the new demand would still be large enough to maintain the high occupancy state.

A rough model for this can be the following. Consider the system of equations

$$\begin{aligned} c'(t) &= \frac{d^m}{k^m + d^m} - c \\ d'(t) &= \frac{M}{p} c(t)\sigma(t) \;-\; d, \end{aligned} \tag{1}$$

where $c(t) \in [0,1]$ is the occupancy rate, $d(t)$ is the demand for the restaurant, $m > 1$, p is the average price charged, and $\sigma(t)$ is a noise term which fluctuates around an average of 1. M is a 'fudge factor' that can be interpreted in terms of the overall quality of the restaurant and the competition from other restaurants.

The sigmoidal nonlinearity $d^m/(k^m+d^m)$ implies that the occupancy rate saturates for large enough d, and that if the demand is below a certain threshold, the occupancy essentially vanishes. For $\sigma \equiv 1$, an input-output analysis (of which there is plenty in our research group) shows that this system has 3 equilibria for sufficiently large M/p: two stable equilibria, including the origin, and one unstable equilibrium.

Now let $\sigma(t)$ fluctuate around 1 over time. Note that the term $(M/p)c(t)\sigma(t)$ can be seen as the 'potential demand', i.e. the value towards which the demand would converge if $c(t)\sigma(t)$ were to stay constant. For very large values of M/p, the non-zero stable equilibrium has a high value for the demand. This doesn't increase the equilibrium value of c, which continues to stay close to 1. But a high equilibrium demand value makes the system less sensitive to changes in σ, in the sense that a stronger decrease in σ would be necessary in order to take the state out of its current basin of attraction.

Interestingly, it seems that if the system starts close to the origin, then an increase in σ is unlikely to induce a switch. In a sense, a noise-induced switch in this system tends to be only from the high demand equilibrium towards the origin.

In short, keeping the constant M/p high, and thus the price low, increases the demand to protect the restaurant occupancy from unforseen eventualities. The artificially high demand creates long lines and forces reservations long time in advance, q.e.d.

What is interesting about the model is its similarity to a simple gene expression model, in which a protein cooperatively induces its own mRNA expression, and where a noise term is involved (in the protein production rate, unfortunately, rather than in the usually noisier mRNA expression). The conclusion regarding the restaurant is straightforward enough that one could have reached it without any model, so that this comparison might actually help explain the quantitative model (which could otherwise be somewhat unintuitive). In this context, one would say that the restaurant owner is paying a price to make their system *robust*.

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